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Abstract

Comparing a fund or any portfolio's performance to a benchmark usually involves risk analysis and attribution analysis. Risk analysis considers measures such as beta and Jensen's alpha to determine if the portfolio is riskier than the benchmark. Attribution analysis decomposes the spread between the portfolio and the benchmark returns into components due to allocating the portfolio sector weights and selecting individual assets differently than the benchmark. These two types of analysis are typically done independently with attribution analysis essentially assuming that there are no differences in risk between the portfolio and the benchmark. As a result, selection and allocation are applied to what is sometimes referred to as "simple or nominal alpha" that is just the difference between the portfolio and benchmark return ignoring risk. Some attempts have been made to combine the two; but none of them have done this in a way that is based on having attribution analysis apply to either Jensen's alpha (using beta as a risk adjustment) or Fama's beta which relies only on relative standard deviations and adjusts for any additional risk due to not having a well-diversified portfolio. This article proposes a risk-adjusted performance attribution analysis that integrates risk measures with the Brinson models of attribution which allows us to decompose the excess portfolio return into components of risk, allocation, selection and net selectivity that is additive and consistent with financial theory. Risk-adjusted performance attribution can give us a quite different interpretation of which sectors contributed to better or worse performance relative to the benchmark. Traditional attribution analysis could result in a manager appearing to have done well in a sector where the higher return relative to the benchmark was just due to taking on more risk. Or the manager could appear to have underperformed in a sector that was a less risky sector. This is demonstrated using actual but masked open-end fund data as the manager portfolio compared to the NCREIF Property Index as the benchmark portfolio.

Some investors are oriented towards long-term nominal or real return goals and therefore don't require the use of benchmarks. Most investors, however, show portfolio returns relative to a benchmark and the portfolio's over or underperformance. This over or underperformance is likely presented on a nominal basis, meaning it does not consider the risk of the portfolio relative to the benchmark. This general practice implicitly assumes that the benchmark has a similar style and risk as the portfolio. Performance attribution analysis that uses this approach can result in misleading conclusions because even though the manager may have beat the benchmark, they may have taken on additional risk by either selecting riskier properties within a sector or allocating more of the portfolio to a sector that is riskier than the benchmark. Conversely, the manager may have selected less risky properties or allocated less of the portfolio to a sector with less risk than the benchmark.

The Capital Asset Pricing Model (CAPM) was introduced by Treynor (1961), Sharpe (1964), Lintner (1965), and Mossin (1966), building on the earlier work of Harry Markowitz (1952) on diversification and modern portfolio theory. In 1968, Jensen's alpha was introduced to measure mutual fund managers' performance. In 1985 and 1986, the Brinson, Hood and Beebower (1985) (hereinafter "Brinson") performance attribution models were introduced to evaluate managers' investment decision making skills. Ankrim (1992) attempted to combine both risk measures and performance attribution, however fell short of reconciling to Jensen's alpha. Subsequently there have been a variety of attempts by Kophamel (2003), Obeid (2005), Menchero (2006/2007), Bacon (2008) and Spaulding (2016); however, all of them fail to incorporate both market risk and unsystematic risk in a way that is additive and that can reconcile to the CAPM and Fama's (1972, 558) concept of "net selectivity" or what we call "Fama's alpha." Thus, to date, no one has successfully combined attribution with a theoretically correct risk adjustment that can include both systematic and unsystematic risk, into one robust model that is intuitive, simple to implement, and provides meaningful results.

This article introduces risk-adjusted performance attribution that builds on the work of all the previously mentioned authors by incorporating beta to calculate risk-adjusted returns for each sector and optionally using Fama's (1972, 558) concept of "net selectivity" to calculate the unsystematic risk portion for portfolios that are less well diversified than the benchmark. The appropriate risk-adjusted returns are then used as inputs into the Brinson models to calculate the allocation, selection (and interaction) components of attribution analysis.

Brinson Attribution

Brinson attribution models are widely used to evaluate performance by decomposing a manager's active return into three main components: allocation, selection and interaction. These components represent the manager's active return contribution for each sector and the weighted sum of each sector's contribution to active return will add to the portfolio's over or underperformance. Brinson attribution helps answer the questions; 1) Did the manager overweight good performing sectors, overweight bad performing sectors, underweight good performing sectors and/or underweight bad performing sectors? 2) How well did the manager select individual investments? 3) Did the manager play up his/her strengths or play down his/her weaknesses? To implement this in-depth analysis, only the weights and returns are needed at the sector levels for both the portfolio and the benchmark. To do multi-period analysis, this information is needed every measurement period, for example, either monthly or quarterly.

Most practitioners, if not all, use Brinson attribution models with nominal returns for the manager's portfolio. As noted above, the results can be very misleading because the risk of the portfolio can differ significantly from the risk of the benchmark. For an apples-to-apples comparison, we need to adjust the risk of the portfolio to be the same as the risk of the benchmark because the purpose of attribution analysis is to explain the difference in return between a fund or portfolio and its benchmark. Thus, we need to "risk-adjust" the portfolio return, due to taking on more (or less) risk than the benchmark. The benchmark risk premium over the risk-free rate is removed when we subtract the benchmark return from the portfolio return as part of the Brinson attribution analysis. We propose an approach that uses beta to calculate risk-adjusted returns and then use what Tofallis (2006, 1365) refers to as a "beta estimator" and Bacon (2013, 89) refers to as "Fama's beta" to further adjust the returns for non-diversification to arrive at what Fama (1972, 558) refers to as "net selectivity."[1]

The CAPM model expresses the expected return for a portfolio (or any asset) as follows:

$$R_F + (R_M - R_F) \times \beta_P \tag{1}$$

where R_F = Risk Free Rate, R_M = Market Return, β_P = Beta of Portfolio

In practice, a suitable benchmark is used as a proxy for the market returns, thus the expected return for a portfolio can be expressed as follows;

$$R_F + (R_B - R_F) \times \beta_P \tag{2}$$

where R_B = Overall Benchmark Return

The difference between the portfolio's actual return and the portfolio's expected return is Jensen's alpha (Portfolio Actual Return – Portfolio Expected Return). The CAPM expected portfolio return is the benchmark return adjusted for the portfolio's beta. Jensen's alpha is therefore the spread after removing all the systematic risk. Jensen's alpha is sometimes referred to as the abnormal return.

Simple or nominal alpha is the actual return relative to the market (benchmark) without taking risk into account and is calculated as the Portfolio Actual Return – Benchmark Return. The difference between nominal alpha and Jensen's alpha is therefore due to taking on additional market risk (risk that is above or below the benchmark risk with a beta of 1). Stated differently, Actual Return – Benchmark Return – Additional Market Risk Premium = Jensen's alpha.

[1] The term "net selectivity" is a somewhat of a misnomer because it is what can be broken down into selection vs. allocation when used in the Brinson attribution analysis. We prefer to refer to it as "Fama alpha."

Risk-Adjusted Performance Example

The following assumptions are provided;

Portfolio Return = 10.5%, Benchmark Return = 6.0%, Portfolio Historical Beta = 1.3, and a Risk-free Rate = 1.0%.

The nominal alpha (portfolio return less benchmark return) is 4.5% but part of that return is due to taking on higher risk with a beta of 1.3 compared to the benchmark beta of 1.0. Using Equation II, based on the portfolio's beta, the portfolio should have earned an expected return as follows:

1.0% + (6.0% - 1.0%) x 1.3 = 7.5%.

Thus, Jensen's alpha is 10.5% - 7.5% = 3.0% which is the amount the manager beat the benchmark after adjusting for risk.

An important question for the purpose of this article is to ask what the manager would have earned *if their portfolio had the same risk as the benchmark* of 1.0 instead of 1.3. In the above example, the nominal alpha is 4.5% but Jensen's alpha is 3.0%. The difference of 1.5% is due to the additional market or beta risk. If we reduce the manager's return of 10.5% by this additional beta risk, we have a risk-adjusted portfolio return of 9.0%. We can arrive at the same risk-adjusted return as follows:

$$R_P - (R_B - R_F) \times (\beta_P - 1) \qquad (3)$$

where R_P = Portfolio Return

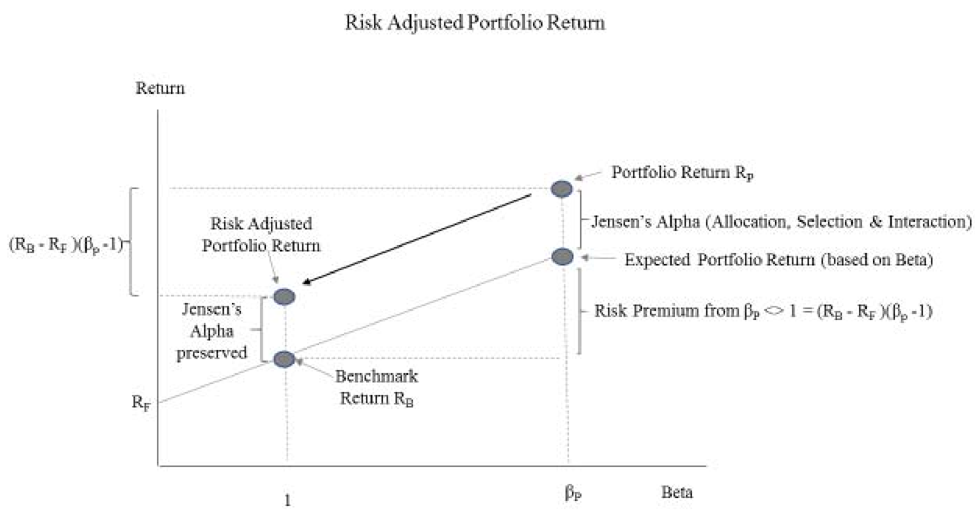
10.5% - (6.0% - 1.0%) x (1.3 – 1.0) = 9.0%

It is this risk-adjusted return that should be used instead of the nominal portfolio return when doing attribution analysis. Note that by "risk-adjusted" we mean adjusting to have a risk *equal to that of the benchmark*. The benchmark still has a risk with a beta of 1.0.

Therefore, for Brinson attribution we need to substitute the portfolio's nominal return with the portfolio's risk-adjusted return. It should also be noted that the difference between the risk-adjusted portfolio return and the benchmark return is Jensen's alpha. In the above example, the risk adjusted return of 9.0% less the benchmark's return of 6.0% is Jensen's alpha of 3.0%. So, by risk adjusting the return, we are removing the additional beta risk while preserving the alpha. That is, we assume that the manager would have the same alpha even if they had the same beta as the benchmark. Any alpha that was earned by the manager is assumed to be the result of an active investment strategy such as overweighting certain sectors at the right time or superior acquisition, disposition and management or properties. We want to decompose any alpha (positive or negative) with the Brinson attribution analysis after adjusting for beta.[2]

Exhibit 1 illustrates how the return is risk-adjusted. The far-right vertical line that crosses the x-axis at the portfolio's beta represents the expected return formula. The top dot is the actual portfolio return (R_P). The bottom dot is the expected portfolio return which is determined by the benchmark return adjusted for the portfolio's beta. The difference between the actual portfolio return and the expected portfolio return is Jensen's alpha. The additional market risk premium resulting from the portfolio's beta greater than or less than the benchmark beta of 1, is the difference between the nominal alpha and Jensen's alpha. The left vertical line that crosses the x-axis at the beta of 1, represents the portfolio risk-adjusted return less the benchmark return that is used in the Brinson model.

Exhibit 1: Risk-Adjusted Portfolio Return



Note that the difference between the risk-adjusted portfolio return and the benchmark return is Jensen's alpha. Risk adjusting the return simply determines what the portfolio return would be if it had a beta of 1 but the portfolio still earned Jensen's alpha. It is the risk-adjusted return that is then used in one of the Brinson attribution models which can be used to calculate the allocation and selection component (and interaction) for each sector. As we will see, the beta risk adjustment applies both to the portfolio and benchmark sector returns, so allocation,

[2]We could also think of preserving alpha as having a hedging strategy to create "portable alpha", although this might also involve hedging costs that might offset some of the alpha. Although our model preserves alpha, an adjustment to the model can be made if one believes the manager cannot earn the same level of alpha at a different beta. For example, the use of leverage can impact both alpha and beta. An extension of our model could incorporate hedging costs to preserve alpha or assume that it decreases with beta.

selection and interaction components are all affected. (Although the overall benchmark has a beta of 1, this is the average of individual sector betas that may be more or less than 1.)

Comparison of Brinson-Fachler to Brinson-Hood-Beebower

The two Brinson models used in the industry are Brinson-Hood-Beebower (1996, "BHB") and Brinson-Fachler (1995, "BF"). BHB and BF are identical, except for the individual sector allocation scores. The following provides a summary of the math for each.

Sector Selection is calculated as the weighted sum of the difference in sector returns multiplied by the benchmark sector weight:

$$(R_{PS} - R_{BS}) \text{ x } W_{BS} \qquad (4)$$

where R_{PS} = Portfolio Sector Return, R_{BS} = Benchmark Sector Return and W_{BS} = Benchmark Sector Weight

Interaction is calculated as the weighted sum of the difference in sector returns multiplied by the difference in sector weights:

$$(W_{PS} - W_{BS}) \text{x } (R_{PS} - R_{BS}) \qquad (5)$$

where W_{PS} = Portfolio Sector Weight

BHB Sector Allocation is calculated as the weighted sum of the difference in sector weights multiplied by the benchmark sector return:

$$(W_{PS} - W_{BS}) \text{ x } R_{BS} \qquad (6)$$

BF Sector Allocation is calculated as the weighted sum of the difference in sector weights multiplied by the difference in the benchmark sector return less the overall benchmark return:

$$(W_{PS} - W_{BS}) \text{ x } (R_{BS} - R_{B}) \qquad (7)$$

With BHB, positive sector allocation scores are achieved when the portfolio overweights a positive performing benchmark sector and/or underweights a negative performing benchmark sector. With BF, positive sector allocation scores are achieved when the portfolio overweights a sector and the benchmark sector overperforms the overall benchmark, or underweights a sector and the benchmark sector underperforms the overall benchmark return. The total allocation score for both Brinson models will always be the same, only the individual sector allocation scores will vary.

Practical Application of Risk-Adjusted Performance Attribution

The following scenario uses the Brinson-Fachler attribution model since it is generally used by most practitioners in real estate. In this example, we used properties in the NCREIF[3] Property Index (NPI) as our benchmark portfolio. The NPI is the best representation of the market portfolio as it contains properties held in separate accounts, closed-end funds as well as open-end funds. For a realistic but hypothetical portfolio, we created an aggregate of properties that consists of all the open-end funds other than the Open-end Diversified Core Equity index funds (ex-ODCE) in the NPI.

The ex-ODCE funds are generally less diversified than ODCE funds. Some are sector specific and many have a core plus or value-add strategies with higher leverage than ODCE funds. However, since we are aggregating all the properties within the ex-ODCE funds, the overall manager portfolio is somewhat diversified, just not like the benchmark portfolio. It should also be noted that the underlying returns used for the NPI benchmark were calculated using the NCREIF unleveraged property level return formula, therefore the returns for the benchmark are un-leveraged.

For the hypothetical manager portfolio, we purposely used the leveraged returns of the properties in the ex-ODCE funds to further demonstrate how our model adjusts for the use of leverage and related risk in the managers' portfolio. Also note that both the returns for the ex-ODCE funds and for the benchmark are based on appraisals so it is an apples-to-apples comparison. Therefore, the use of ex-ODCE funds in our attribution model will best demonstrate how it adjusts for risk. So, the results are *as if* you invested in each of the ex-ODCE funds which is possible in practice since they are open-end funds and therefore makes a good example to illustrate the attribution analysis without identifying or analyzing any particular investor's account. Some portfolios obviously were above and others below the average returns used in this example.

The data used for this example cover the 5 years ended December 31, 2018. Since this risk-adjusted attribution analysis examines return and risk on an ex-post basis, the beta shown represents actual beta for the same 5-year performance measurement period being evaluated. Beta, like any other return or risk measure can vary depending on the time horizon analyzed. Historical beta, just like past returns, may not be a reliable indicator of future results. While the expected beta at the start of the measurement period may have been different than the actual beta realized during the measurement period, we believe that the actual beta for the same measurement period should be used to risk-adjust the returns for performance measurement purposes. The property counts for the hypothetical portfolio and the NPI benchmark at December 31, 2018 were 763 and 7,883 respectively, representing a gross market value of $63.3 billion and $608.7 billion. Exhibit 2 is the typical data set used to perform the analysis.

[3] The National Council of Real Estate Investment Fiduciaries (NCREIF) is a leading private equity real estate index provider in the United States, consisting of investment managers, institutional investors, consultants, appraisers, academics, researchers and other industry professionals. More information is available at www.NCREIF.org.

Exhibit 2: Data Set for Brinson Attribution Analysis

Sector	Portfolio Weights	Benchmark Weights	Portfolio Returns	Benchmark Returns	Nominal Alpha
Apartment	23.3%	24.5%	12.4%	8.3%	
Hotel	0.2%	1.2%	0.4%	8.2%	
Industrial	18.9%	14.2%	23.0%	13.6%	
Office	39.2%	36.9%	14.2%	8.8%	
Retail	18.4%	23.2%	10.1%	9.0%	
Total	100.0%	100.0%	14.7%	9.4%	5.3%

The portfolio and the benchmark each consist of five sectors and the allocation weights and returns are shown for both. Without doing any in-depth analysis, the portfolio outperformed the benchmark resulting in an active return or nominal alpha of 5.3%. The manager outperformed the benchmark in all sectors, except hotel, and overweighted industrial and office, and underweighted the other three sectors.

Exhibit 3: Calculate Attribution Components for Nominal Alpha

Sector	BF Allocation	BF Selection	BF Interaction	*Nominal Alpha*
Apartment	0.0%	1.0%	-0.1%	0.9%
Hotel	0.0%	-0.1%	0.1%	0.0%
Industrial	0.2%	1.4%	0.4%	2.1%
Office	0.0%	2.0%	0.1%	2.1%
Retail	0.0%	0.3%	-0.1%	0.2%
Total	**0.2%**	**4.6%**	**0.5%**	**5.3%**

In Exhibit 3 the active return or nominal alpha of 5.3% is due to a positive allocation score of 0.2%, a positive selection score of 4.6% and a positive interaction score of 0.5%. It was office and industrial that contributed 2.1% each to the overall score, followed by apartment with a score of 0.9%.

Risk Adjustments

The above traditional attribution analysis may be misleading because it only incorporates weights and returns. It does not consider whether or not the manager took on more or less risk than the benchmark. The manager could have selected riskier properties in a sector or allocated more of the portfolio to riskier sectors. Risk-adjusted returns must be used in order to evaluate both returns and risk together.

The risk analysis may be different depending on how risk is defined and how it is priced. For example, if you have a diversified portfolio (or assume the ultimate investors will have a diversified portfolio), you could just focus on market risk, otherwise known as systematic risk because unsystematic or idiosyncratic risk can be diversified away. If, however; your portfolio is not diversified, or you simply believe that the standard deviation is a more relevant risk measure, you could focus on total risk that combines both systematic and unsystematic risk. We will demonstrate this in two separate steps, first an analysis of systematic risk based on the CAPM and then an analysis of total risk that is based only on standard deviations as a risk measure and thus does not rely on the validity of the CAPM.

The price per unit of risk as defined by CAPM can be calculated as the market excess return divided by the beta of the market return. This is the Treynor Ratio of the benchmark. By definition, the beta of the benchmark is 1.0 so the price of risk is just the benchmark excess return (benchmark return minus the risk-free rate). The expected excess return for any portfolio is then this price of risk multiplied by the beta for the portfolio. That is, we have

$$R_P - R_F = \beta_P \times (R_M - R_F) \quad (8)$$

In practice, a benchmark portfolio is used as a proxy for the market portfolio when calculating beta. Understanding how beta is used to measure risk is important to understanding the different risk-adjusted attribution models we are proposing, one model that adjusts for systematic risk (beta) and another model that adjusts for a more comprehensive risk (total risk or standard deviation) that includes both systematic and unsystematic risk in a way that is additive to beta.

In our initial model we are pricing risk consistent with CAPM and therefore can reconcile to Jensen's alpha. Others have proposed risk-adjusted attribution models that price risk differently. For example, Spaulding (2016, 51) uses M^2 where risk is defined as standard deviation and it is priced using the manager portfolio's excess returns rather than the benchmark return. If the Treynor ratio was used instead of M^2, risk would be defined as beta rather than the standard deviation, but it would still be priced based on the manager portfolio excess returns. Because these other models price risk based on the portfolio excess returns and not the benchmark excess returns, the results will not reconcile to CAPM. Our model prices risk based on the benchmark excess returns, so whether beta or standard deviation is used, the model will measure the amount of risk in a consistent manner such that the unsystematic and systematic risk are additive.

The data necessary to risk-adjust the portfolio returns for systematic or beta risk are shown in Exhibit 4. It is important to note that the systematic risk for each sector must be measured relative to the overall benchmark return (weighted average of each sector) and not the benchmark return for each sector. Systematic risk is measured relative to the "market" portfolio with the benchmark a proxy for the market. In this way, the overall beta is the weighted average of the beta for each sector as it should be. It should also be noted that even though the overall benchmark return is 1.0 by definition, some sectors within the benchmark may have a beta greater than 1 with others less than one, so the weighted average is 1.0. Thus, we need to risk adjust both the manager's portfolio and the benchmark sectors.

Exhibit 4: Data Necessary to Beta Risk-Adjust the Portfolio and Benchmark Sector Returns

Portfolio Sector	Portfolio Returns	Correlation with the Overall Benchmark Return	Std Dev Portfolio Excess Returns	Std Dev Overall Benchmark Excess Returns	Risk Free Rate	Portfolio Beta	Overall Benchmark Return	Portfolio Risk Adjusted Return
Apartment	12.4%	0.782	2.87%	1.35%	1.0%	1.665	9.4%	6.8%
Hotel	0.4%	0.956	6.23%	1.35%	1.0%	4.412	9.4%	-28.2%
Industrial	23.0%	-0.027	2.00%	1.35%	1.0%	-0.040	9.4%	31.7%
Office	14.2%	0.805	3.71%	1.35%	1.0%	2.214	9.4%	4.1%
Retail	10.1%	0.876	3.22%	1.35%	1.0%	2.091	9.4%	1.0%
Total (weighted average)	**14.7%**	**0.712**	**3.11%**	**1.35%**	**1.0%**	**1.642**	**9.4%**	**9.3%**

Benchmark Sector	Benchmark Returns	Correlation with the Overall Benchmark Return	Std Dev Benchmark Excess Returns	Std Dev Overall Benchmark Excess Returns	Risk Free Rate	Benchmark Beta	Overall Benchmark Return	Benchmark Risk Adjusted Return
Apartment	8.3%	0.953	1.15%	1.35%	1.0%	0.814	9.4%	9.9%
Hotel	8.2%	0.614	2.29%	1.35%	1.0%	1.043	9.4%	7.9%
Industrial	13.6%	0.366	0.63%	1.35%	1.0%	0.172	9.4%	20.6%
Office	8.8%	0.959	1.47%	1.35%	1.0%	1.044	9.4%	8.4%
Retail	9.0%	0.895	2.54%	1.35%	1.0%	1.688	9.4%	3.2%
Total (weighted average)	**9.4%**	**0.881**	**1.53%**	**1.35%**	**1.0%**	**1.000**	**9.4%**	**9.3%**

Each sector's actual return for both the portfolio and the benchmark are de-risked or risk-adjusted for the amount of beta risk greater than or less than 1. The formula to risk-adjust the portfolio returns was shown in Equation 3. The formula is used for the sector returns. The benchmark sector returns are risk-adjusted in a similar manner substituting the Benchmark Return for the Portfolio Return as follows;

$$R_{BS} - [(R_B - R_F) \times (\beta_{PS} - 1)] \quad (9)$$

where R_{BS} = Benchmark Sector Return, β_{PS} = Beta of the Portfolio Sector

For example, the benchmark's apartment risk-adjusted return is;

8.3% - [(9.4% - 1.0%) x (0.814 – 1)] = 9.9%

Notice the risk-adjusted benchmark apartment return is actually higher than the nominal return since its beta is less than 1. The risk-adjusted returns for both the portfolio and the benchmark can now be substituted in the Brinson model for the actual returns. The basic data necessary and the resulting beta risk-adjusted performance attribution is shown in Exhibit 5.

Exhibit 5: Risk-Adjusted Performance Attribution Using Beta

Sector	Portfolio Weights	Benchmark Weights	Risk-Adjusted Portfolio Returns	Risk-Adjusted Benchmark Returns
Apartment	23.3%	24.5%	6.8%	9.9%
Hotel	0.2%	1.2%	-28.2%	7.9%
Industrial	18.9%	14.2%	31.7%	20.6%
Office	39.2%	36.9%	4.1%	8.4%
Retail	18.4%	23.2%	1.0%	3.2%
Total	**100.0%**	**100.0%**	**9.3%**	**9.3%**

Sector	Risk Adjusted BF Allocation	Risk Adjusted BF Selection	Risk Adjusted BF Interaction	Jensen's Alpha
Apartment	0.0%	-0.8%	0.0%	-0.8%
Hotel	0.0%	-0.4%	0.4%	0.0%
Industrial	0.5%	1.6%	0.5%	2.6%
Office	0.0%	-1.6%	-0.1%	-1.7%
Retail	0.3%	-0.5%	0.1%	-0.1%
Total	**0.8%**	**-1.7%**	**0.9%**	**0.0%**

As shown in Exhibit 6, the risk-adjusted scores calculated in Exhibit 5 can now be compared to the nominal attribution scores previously calculated in Exhibit 3.

Exhibit 6: Calculate Market Risk as Nominal Alpha Less Jensen's Alpha

Sector	Nominal Alpha	Jensen's Alpha	Market Risk
Apartment	0.9%	-0.8%	1.7%
Hotel	0.0%	0.0%	0.0%
Industrial	2.1%	2.6%	-0.6%
Office	2.1%	-1.7%	3.9%
Retail	0.2%	-0.1%	0.3%
Total	**5.3%**	**0.0%**	**5.3%**

On a nominal basis, the portfolio overperformed the benchmark by 5.3%, however on a systematic market risk-adjusted basis the portfolio return equaled the benchmark return, primarily driven by office and apartment underperformance, offset by industrial overperformance. As noted in Exhibit 6, outside of hotel, office had the highest beta and industrial had the lowest, which impacted their market risk of 3.9% and (0.6%), respectively. This highlights what happens when picking investments that are more or less risky than the benchmark.

The approach discussed above to risk-adjust returns is done for each sector for the attribution analysis. This can be done using either the BF or BHB versions. We prefer BF for the following reasons: First, BF provides more insight into the interpretation of the allocation component for individual sectors without impacting the selection component. For example, the allocation score will be positive if a sector is overweighted that has a benchmark sector return that is higher than the overall benchmark return. The BHB model only requires the sector benchmark return be positive to have a positive allocation score for the overweighted portfolio sector. Our risk-adjusted attribution approach complements BF nicely by providing a better interpretation of the allocation, selection and interaction scores for each sector. For example, the selection component might be positive for a sector before risk-adjusting but lower or negative after risk-adjusting the sector.

Adjusting Alpha for Diversification

Fama (1972, 557-558) shows how to further decompose Jensen's alpha into the portion that is due to having less diversification than the benchmark. As he pointed out, this is especially relevant if the portfolio is the only one held by the investor and thus the only source of diversification. The idea is that in addition to being compensated for beta (systematic) risk, the investor needs compensation for taking on more unsystematic risk by deviating from the benchmark which is presumably well diversified. The required return for taking on this additional risk is the risk premium for non-diversification, calculated as follows:

$$(R_B - R_F) \times (\beta_F - \beta_P) \qquad (10)$$

where β_F has been referred to as "Fama's beta." Fama's beta can be calculated by taking the regular beta and dividing by the correlation coefficient. In effect, this removes any benefit of diversification that was in the original beta such that it is capturing the total risk relative to the benchmark. Alternatively, Fama beta is simply the ratio of the standard deviation of the portfolio to the standard deviation of the benchmark which also shows how it is capturing total risk relative to the benchmark. Because it is treated as if it was a beta, it becomes additive to the CAPM that only captures systematic risk.

The expected return with this additional risk adjustment is therefore as follows:

$$R_F + (R_B - R_F) \times \beta_P + (R_B - R_F) \times (\beta_F - \beta_P) \tag{11}$$

The risk premium is shown in two components with the first being due to the CAPM beta and the second due to the incremental risk premium for lack of diversification based on the difference between Fama beta and CAPM beta. The difference between the actual portfolio return and this expected return is referred to as "net selectivity" which includes a more comprehensive risk adjustment that might be relevant for many investors. To get the more comprehensive Fama risk-adjusted return, we now have;

$$(R_B - R_F) \times (\beta_F - 1) \tag{12}$$

where Fama's beta β_F is used in place of the regular beta

The more comprehensive portfolio risk-adjusted return can now be substituted for the nominal portfolio return in the Brinson attribution analysis in the same way that was discussed previously with the beta risk-adjusted return.

The incorporation of this total risk adjustment in the model is incorporated in Exhibit 7. The slope of the line is the same as the equation for expected return using regular beta and using Fama's beta, as illustrated;

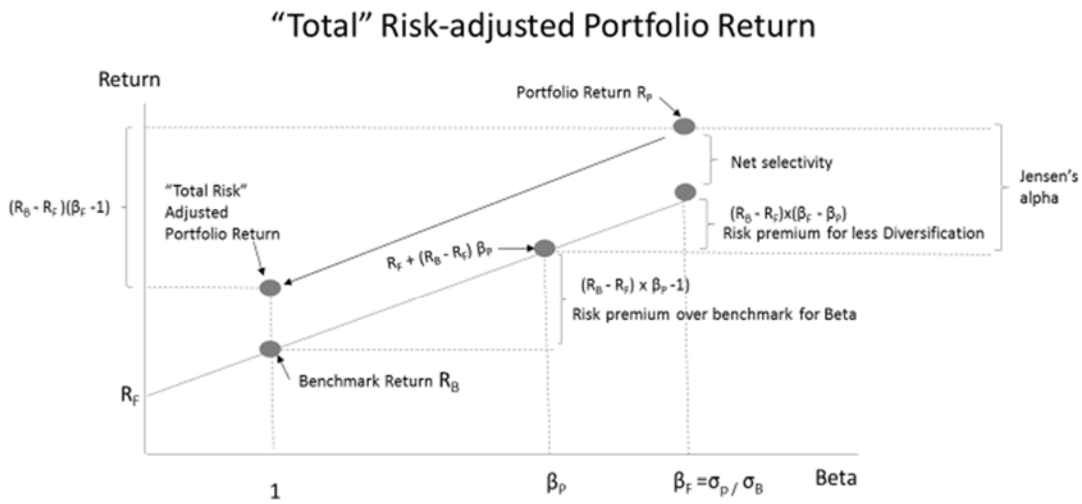
$$\text{CAPM Expected Return} = R_F + (R_{BE} \times \beta_P) \tag{13}$$

$$\text{Fama Expected Return} = R_F + (R_{BE} \times \beta_F) \tag{14}$$

The Risk-free rate (R_F) is the y-intercept and the slope of both lines is the Benchmark Excess Return / beta, where beta is 1 for the benchmark. Fama's beta is further out on the risk spectrum when the correlation is less than 1. The line extends by the additional risk premium for unsystematic risk which is shown in Equation 7.

If the correlation is equal to 1, then Fama's beta and regular beta are one and the same. The resulting risk-adjusted return will be lower than it would have been by lowering the return in parallel with the regular beta that has a smaller slope as was illustrated in Exhibit 1.

Exhibit 7: "Total" Risk-Adjusted Portfolio Return (including Unsystematic Risk)

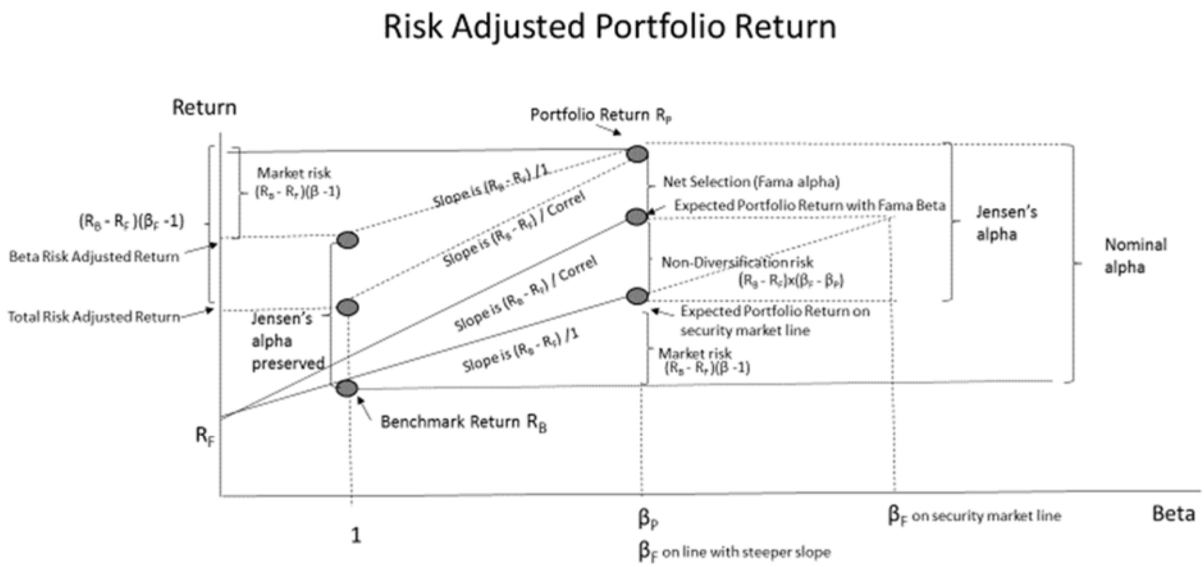


As noted above, the difference between the regular beta that reflects systematic risk and the use of Fama beta that includes the impact of having less diversification depends on the correlation between the portfolio and the benchmark. Fama beta is the regular beta divided by the correlation coefficient. If the two were perfectly correlated, there would be no difference because this would mean there is no difference in diversification. The lower the correlation, the greater the difference because the portfolio is lesser and lesser diversified. We can decompose the total risk portfolio expected return as follows:

$$R_F + R_{BE} + R_{BE} \times (\beta_P - 1) + R_{BE} \times (\beta_F - \beta_P) \qquad (15)$$

It may be useful to contrast to two approaches for getting a risk adjusted return to use in the attribution analysis. Exhibit 8 combines the two approaches in the same exhibit. To do this we use a slightly different approach to represent the different betas (CAPM systematic risk vs. Fama total risk). When total risk is included in the risk premium, we have a line that has a steeper slope than the CAPM security market line based on beta. The steeper slope is $(R_M – R_F)$ / Correl (R_P, R_B). With a correlation coefficient less than one, the slope will be steeper than the security market line. The term $(R_M – R_F)$ / Correl (R_P, R_B) is the price of total risk with the regular beta being the amount of risk. Rather than dividing beta by the correlation coefficient to get the Fama beta, we can divide $(R_M – R_F)$ by the correlation coefficient and use the CAPM beta as the amount of risk. This is simply a different way of representing the same risk premium. But it allows us to contrast the two risk premiums as shown in Exhibit 8. Risk adjusting the portfolio return for total return shifts the return down along the steeper slope which results in a greater risk adjustment and a lower risk adjusted return. Note that the Fama beta can be interpreted two ways in the diagram. It is the same as the regular CAPM beta when the expected return is on the higher slope line. It is further to the right based on the traditional security market line. The level of expected return is the same in either case. The difference as noted above is simply how we think of the price of risk vs. the amount of risk.

Exhibit 8: Combined Approach for Risk-Adjusted Portfolio Returns



Example Extended for Unsystematic (Standard Deviation) Risk

If we followed the same steps in calculating beta risk-adjusted attribution and substituted Fama beta for beta, we would have the results on a total risk-adjusted basis, one that includes both market and unsystematic risk, the later, being the risk premium for not being diversified like the benchmark. Fama beta for the sectors are the standard deviations of the sector excess returns over the risk-free rate divided by the standard deviation of the overall benchmark using the weighted average of the benchmark sector standard deviations, which will differ from the overall standard deviation of the benchmark returns in total. Specifically, notice that the weighted average standard deviation of 1.53% in Exhibit 9 is higher than the overall benchmark standard deviation of 1.35% shown in Exhibit 4. Exhibit 9 shows a Fama risk-adjusted portfolio return for apartments as 5.0%, calculated as follows;

$$R_{PS} - R_{BE} \text{ x } (\beta_F - 1) \quad (16)$$

12.41% - [(9.4% - 1.0%) x (1.876 - 1)] = 5.0%

Exhibit 9: Fama Risk-adjusted Returns

Sector	Portfolio Returns	Std Dev Portfolio Excess Returns	Std Dev Benchmark Excess Returns (wtd avg)	Risk Free Rate	Fama Beta	Overall Benchmark Return	Fama Risk Adjusted Return
Apartment	12.4%	2.87%	1.53%	1.0%	1.876	9.4%	5.0%
Hotel	0.4%	6.23%	1.53%	1.0%	4.070	9.4%	-25.4%
Industrial	23.0%	2.00%	1.53%	1.0%	1.305	9.4%	20.4%
Office	14.2%	3.71%	1.53%	1.0%	2.425	9.4%	2.2%
Retail	10.1%	3.22%	1.53%	1.0%	2.103	9.4%	0.8%
Total (weighted average)	**14.7%**	**3.11%**	**1.53%**	**1.0%**	**2.029**	**9.4%**	**6.0%**

As we can see, the total portfolio Fama risk-adjusted return is 6.0%. If we subtract the this from the beta risk-adjusted portfolio return of 9.3% we calculated in Exhibit 5, we get a non-diversification risk premium of 3.3%. We can easily check this total portfolio non-diversification risk premium as follows;

$$(R_{BE}) \text{ x } (\beta_F - \beta_P)$$

$$(9.4\% - 1.0\%) \text{ x } (2.029 - 1.642) = 3.3\%$$

Now that we have explained all the calculations, a summary of the risk-adjusted attribution results can be presented as shown in Exhibit 10.

Exhibit 10: Summary of Risk-Adjusted Performance Attribution

Risk Adjusted Attribution Analysis	Allocation	Selection	Interaction	Total
Nominal Alpha	**0.2%**	**4.6%**	**0.5%**	**5.3%**
Market Risk	-0.6%	6.3%	-0.4%	5.3%
Jensen's Alpha	**0.8%**	**-1.7%**	**0.9%**	**0.0%**
Non-Diversification Risk	0.0%	2.8%	0.5%	3.3%
Fama Alpha (Net Selectivity)	**0.8%**	**-4.5%**	**0.4%**	**-3.3%**

Detail Risk-adjusted Attribution by Sector

Component	Apartment	Hotel	Industrial	Office	Retail	Total
Nominal Allocation	0.0%	0.0%	0.2%	0.0%	0.0%	0.2%
Nominal Selection	1.0%	-0.1%	1.4%	2.0%	0.3%	4.6%
Nominal Interaction	-0.1%	0.1%	0.4%	0.1%	-0.1%	0.5%
Total Nominal Alpha	**0.9%**	**0.0%**	**2.1%**	**2.1%**	**0.2%**	**5.3%**
Jensen Allocation	0.0%	0.0%	0.5%	0.0%	0.3%	0.8%
Jensen Selection	-0.8%	-0.4%	1.6%	-1.6%	-0.5%	-1.7%
Jensen Interaction	0.0%	0.4%	0.5%	-0.1%	0.1%	0.9%
Total Jensen's Alpha	**-0.7%**	**0.0%**	**2.6%**	**-1.7%**	**-0.1%**	**0.0%**
Fama Allocation	0.0%	0.0%	0.5%	0.0%	0.3%	0.8%
Fama Selection	-1.3%	-0.3%	0.3%	-2.5%	-0.6%	-4.4%
Fama Interaction	0.0%	0.3%	0.1%	-0.2%	0.1%	0.3%
Total Fama's Alpha	**-1.3%**	**0.0%**	**0.9%**	**-2.7%**	**-0.2%**	**-3.3%**

In summary, the portfolio generated a 14.7% return, beating the benchmark by 5.3%, without adjusting for risk. This significant overperformance is due to the manager taking on more risk than the benchmark. You will recall that the manager portfolio reflects leveraged returns of the properties in the ex-ODCE open-end funds, whereas the NPI benchmark includes property performance on an unleveraged basis. Therefore, the methodology we used adjusted both for the risk due to leverage and any other reason that the manager's portfolio was riskier than the benchmark.

Clearly factoring in risk makes a big difference. The differences between the alphas are the systematic and unsystematic risk of 5.3% and 3.3%, respectively, for a total risk adjustment of 8.6%. Jensen's alpha takes into account the risk that the portfolio has relative to the market or in this case the benchmark beta which is always 1. As noted in Exhibit 4 the portfolio had a beta of 1.642, therefore the portfolio is riskier. Fama's alpha takes into account the total risk which includes both the beta risk and the additional standard deviation risk. As noted in Exhibit 9 the portfolio had a Fama beta of 2.029, therefore the portfolio is riskier on this basis as well.

The details by sector are also presented in Exhibit 10. On a nominal basis before adjusting for risk, office was the largest contributor to the 5.3% overperformance. On a beta and standard deviation risk-adjusted basis, office had the worst negative alpha. The only sector that had positive risk-adjusted performance on both a beta and standard deviation basis was industrial.

Another interesting note is that we also ran our risk-adjusted model using the unleveraged returns of the ex-ODCE funds instead of the leveraged returns as were used above. The unlevered manager portfolio produced a 9.9% return vs. the levered portfolio of 14.7%. You will recall that after adjusting for beta risk the 14.7% levered return was reduced to 9.3%, whereas after adjusting for beta risk the 9.9% unleveraged return was increased to 11.0%. Because the risk-adjusted levered return of 9.3% underperformed the risk-adjusted unlevered portfolio of 11.0%, this indicates a poor use of leverage. The manager may have put too much leverage on the properties at exactly the wrong time, for instance when values were declining. Or the manager may have used variable rate debt rather than fixed rate debt at the wrong time in the cycle.

A Note on Interaction

In addition to allocation and selection components, attribution analysis has an interaction term that is the difference in weights times the difference in returns. Some analysts prefer to combine this term with either allocation or selection rather than showing it separately. Rather than take a position on this debate, we simply want to point out that if it is kept as a separate term, it does have some interpretation in our risk-adjusted attribution analysis. The interaction term will be more positive (or less negative) if the manager has either overweighted a less risky sector or underweighted a riskier sector. Vice versa, the interaction term will be less positive or more negative if the manager has overweighted a riskier sector or underweighted a less risky sector.

Reliability of Risk Measures

The real estate asset class always provides a challenge in measuring risk no matter what measure one thinks is best to use. Most risk measures require periodic returns with sufficient frequency and duration. The challenges of using appraisal-based returns are well known although that issue is somewhat mitigated when both the manager's portfolio and benchmark use appraisal-based returns. Our methodology assumes that one is able to calculate a beta and / or a standard deviation. This is true of most measures of risk for a portfolio whether it is Sharpe ratios, betas, tracking error, etc. Part of the risk is being able to accurately measure the risk. One should always question whether the measures being used in an analysis or risk are statistically significant. The betas or standard deviations are always a single point estimate within a distribution around the "true" measure. So, one should always treat any risk analysis with a "grain of salt." That said, we would suggest that it is better to attempt to measure and adjust for risk rather than just assume that it is the same for a manager's portfolio and the benchmark. We have seen with some real data from the NCREIF database that there can be significant differences in the betas and standard deviations. Over time we expect that the appraisals and related data will continue to improve and methodologies such as we suggested in this paper will become even more reliable and useful to managers and investors.

Conclusion

We have developed a risk-adjusted performance attribution analysis that integrates risk measures with the Brinson models of attribution which allows us to decompose the excess portfolio return into components of risk, allocation and net selection that is additive and consistent with financial theory. Either the beta traditionally used to calculate Jensen's alpha or the beta proposed by Fama to incorporate unsystematic risk and relies only on relative standard deviations to adjust for risk can be used to risk adjust the returns that are incorporated into the Brinson attribution analysis. The result is that the difference between the portfolio and benchmark return can be decomposed into the following components: (1) risk premium due to the portfolio beta, (2) risk premium due to lack of diversification (optional), (3) net selection, (4) allocation, and (5) interaction. The second component can be included if it is believed that investors cannot hold well diversified real estate portfolios, or they simply believe that the standard deviation is a better measure of risk than the beta from the CAPM.

Risk-adjusted performance attribution can give us quite different interpretation of which sectors contributed to better or worse performance relative to the benchmark. Traditional attribution analysis could result in a manager appearing to have done well in a sector where the higher return relative to the benchmark was just due to taking on riskier investments or allocating more to a riskier sector. Or the manager could appear to have underperformed in a sector that was actually a less risky sector or the manager selected less risky investments.

This article has used single period examples to illustrate the concept. But it can easily be extended to a multi-period analysis as is done with traditional attribution analysis.

Although our focus has been on a risk adjustment model that is consistent with the CAPM, it should be noted that beta is simply the sensitivity of a portfolio's return to the benchmark's return. It is the slope of a simple linear regression that explains the portfolio return (or excess return over the risk-free rate) by the benchmark return (or excess return). Thus, even if one questions the validity of the CAPM assumptions, risk adjusting by beta and doing attribution analysis of Jensen's alpha is still relevant. Also, the methodology we proposed can be extended to included more than one beta based on a multi-factor risk model. Furthermore, the use of Fama's beta only relies on the total risk which just uses the standard deviation of the portfolio relative to the benchmark standard deviation. The point is that risk adjustment needs to be formally integrated into attribution analysis as we have done. Our methodology allows the analyst to decompose the risk into different components and get a more meaningful interpretation of the selection and allocation components of attribution analysis while having a choice as to whether to price risk based on beta or the standard deviation and know the relative contribution of each.

Future research could evaluate how the results would differ for other ways of pricing risk. As suggested above, one could explore using a multi-beta model with different risk factors. Alternative models could also be explored that allow for an interaction between alpha and beta or formally introduce hedging costs to preserve alpha. The risk adjusted returns and resulting alphas discussed in this article might also be explored as to how they might be used to create better incentive fee structures where the manager is only rewarded for earning alpha.

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